

Стратегія стане дороговказом при реалізації місцевої політики у сфері цифровізації, впровадженні діджитал-інструментів у сфері надання публічних послуг, розвитку електронної демократії, підвищення ефективності та прозорості діяльності органу місцевого самоврядування.

Таким чином, важливою особливістю цифрового розвитку територіальних громад на сьогодні є адаптація під виклики війни та втілення актуальних цифрових рішень з урахуванням сьогоднішніх потреб населення, направлення вектору цифровізації на відновлення та розвиток громади. У період воєнного стану та повоєнної відбудови актуальним питанням стає пошук для кожної територіальної громади свого шляху до розбудови цифрової спроможності громади та визначення стратегічних цілей розвитку цифровізації громади, впровадження цифрових інструментів для посилення ефективності процесів відбудови та відновлення громади.

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IMPLEMENTATION OF PROBABILITY THEORY IN ECONOMICS ON THE EXAMPLE OF THE LAW OF LARGE NUMBERS

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Probability theory always works with different kind of problems in various spheres of life. In this abstract we are going to explain the work of Maths in practice. For example, there are two maternity hospitals, one small and one large in our city. According to the statistical report, it is stated that 70% of the babies born this year are boys, and 30% are girls. The question is: which hospital do you think this statistic likely belongs to — the small or the large one? At the end of this article, we will answer this question and provide a rationale for it.

The Italian mathematician and engineer Gerolamo Cardano, published his book “De Ludo Aleae”, where he introduces the idea that the accuracy of empirical statistics improves with the number of trials. In other words, the more experiments conducted, the closer the obtained average value is to the true one. This marked the beginning of one of the fundamental law in probability theory — the Law of Large Numbers (LLN). This law allows us to predict the behavior of a random variable, whether it's the average height of a person, the probability of a baby being born male or

female, the average profit a casino makes per round in a given game, the profitability of insurance terms for a company, and so on (Contributors to Wikimedia projects, 2002a).

Informal Definition of the Law of Large Numbers. The arithmetic mean of a large number of independent, identically distributed random variables stabilizes as their number increases (Contributors to Wikimedia projects, 2002b). Let's look at an example. Take a six-sided die and repeatedly roll it, calculating the arithmetic mean of the scores obtained from all rolls after each throw.

Roll	1:	Roll	2:	
Outcome:	6	Outcome:	2	...
Arithmetic mean:	$6/1=6$	Arithmetic mean:	$(6+2)/2=4$	

To automate the simulation of our experiment, let's write a small Python script. We'll visualize the results using the Matplotlib library. You can find source code here:

<https://github.com/YehorSeniuk/LLNDiceRollsModeling/blob/main/main.py>

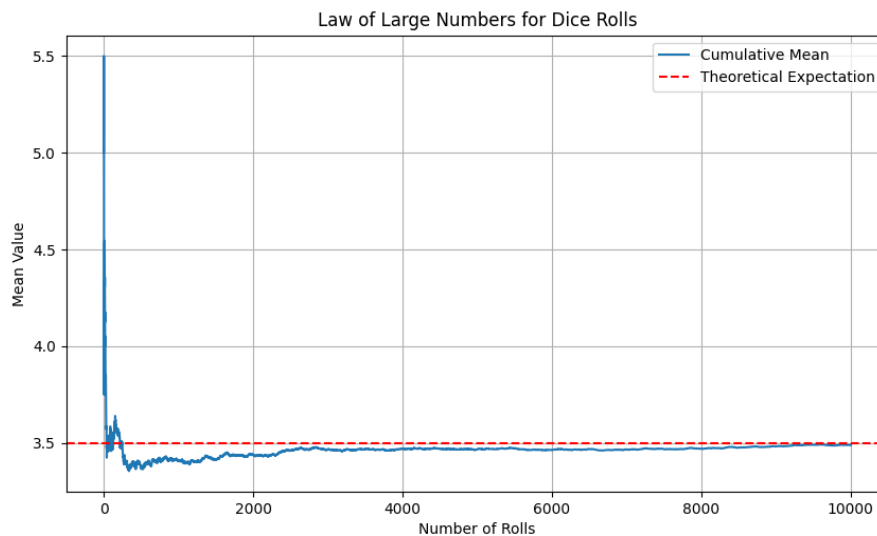


Figure 1 - LLN for Dice Rolls

Formal Definition of the Law of Large Numbers. To begin with, there are two types of the Law of Large Numbers: the Weak Law of Large Numbers (WLLN) and the Strong Law of Large Numbers (SLLN). They differ in terms of the type of convergence and conditions for applicability. The Weak LLN uses convergence in probability, while the Strong LLN uses almost sure convergence. Let's examine each type of convergence separately.

Convergence in Probability. Let's consider a sequence of random variables $(X_i)_{i=1}^n$

The sequence of random variables converges in probability if and only if, as n approaches infinity, for any arbitrarily small positive probability, the probability that the absolute deviation of the arithmetic mean \bar{X} from the expected value exceeds tends to zero. In other words:

$$\lim_{n \rightarrow \infty} P(|\bar{X}_n - \mu| > \epsilon) = 0 \quad (1)$$

Almost Sure Convergence. Let's consider a sequence of random variables $(X_i)_{i=1}^n$

The sequence of random variables converges almost surely if and only if, for almost every possible outcome, the values of the sequence will eventually get arbitrarily close to a certain value as the number of observations increases. In other words:

$$P(\lim_{n \rightarrow \infty} \bar{X}_n = \mu) = 1 \quad (2)$$

Weak Law of Large Numbers. Let's consider a sequence of random variables $(X_i)_{i=1}^n$ where all random variables:

1. Pairwise independent
2. Have identical distribution
3. Have finite variance

Then convergence in probability holds, that is $\lim_{n \rightarrow \infty} P(|\bar{X}_n - \mu| > \epsilon) = 0$.

Strong Law of Large Numbers. Let's consider a sequence of random variables $(X_i)_{i=1}^n$ where all random variables:

1. Pairwise independent
2. Have identical distribution
3. Have finite variance
4. Satisfies $n^{-1}D[\bar{X}_n] \rightarrow 0$ (Kolmogorov theorem)

Then almost sure convergence holds, that is $P(\lim_{n \rightarrow \infty} \bar{X}_n = \mu) = 1$.

It is also important to note that the fulfillment of the Strong Law of Large Numbers implies the fulfillment of the Weak Law of Large Numbers; however, the reverse is generally not true. This is because almost sure convergence of a random variable implies its convergence in probability, but the reverse is not generally true, that is:

$$P(\lim_{n \rightarrow \infty} \bar{X}_n = \mu) = 1 \Rightarrow \lim_{n \rightarrow \infty} P(|\bar{X}_n - \mu| > \epsilon) = 0 \quad (3)$$

Application. Insurance. Suppose we are opening an insurance company. We face the question: how can we determine whether a particular insurance offer is profitable for the company, considering the frequency of claims, the cost of insurance, and the payout in the event of a claim in the long run? To answer this question, let's recall the Law of Large Numbers. Thus, if the random variable that describes the insurance claim meets the criteria of the Law of Large Numbers, we can expect that empirical statistics, such as the arithmetic mean of income and expenses, will tend toward its true value, that is, its expected value. Now, all we need to do is calculate this expected value based on the probability of a claim occurring (obtained from statistics).

Let the random variable X_i represent the income or expense in the i -th insurance case, then

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i \quad (4)$$

The Promised Answer. Most likely, this statistic belongs to the smaller hospital, and here's why: it is a statistical fact that boys and girls are born with approximately equal probability, that is, $P(X=M) = P(X=G) = 0.5$. The report indicated that 70% of those born are boys, which is a significant deviation (we would even call it a leap) from the overall statistics; this, in turn, is characteristic of small samples. Indeed, let's recall how the arithmetic mean behaves when calculating scores during dice rolling.

The Law of Large Numbers is one of the fundamental laws of probability theory. It has established a connection between the number of experiments and the accuracy of the obtained values, thereby making an invaluable contribution to the experimental sciences, such as physics, astronomy, medicine, and many others. It is also worth noting its significant impact on the development of fields such as economic sciences, insurance sphere, illegal gaming and others.

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